Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Sound and Vibration 268 (2003) 632–634
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Letter to the Editor

# Comments on "Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping"

R. Van Dooren*

*Department of Mechanical Engineering, Free University of Brussels, Pleinlaan 2, B-1050 Brussels, Belgium*

Received 20 January 2003

In Ref. [1] Chen investigated the dynamic behavior of a symmetric gyro with linear-plus-cubic damping. The gyro is mounted on a vibrating base represented by a periodic excitation with the amplitude $f$ and the frequency $\omega$. With the definitions $x_1 = \theta$, $x_2 = \dot{\theta}$, the equation of motion governing the nutation $\theta$ of the gyro is given by [1]

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -\alpha^2(1 - \cos x_1)^2/\sin^3 x_1 - c_1 x_2 - c_2 x_2^3 + \beta \sin x_1 + f \sin x_1 \sin \omega t. \quad (1)$$

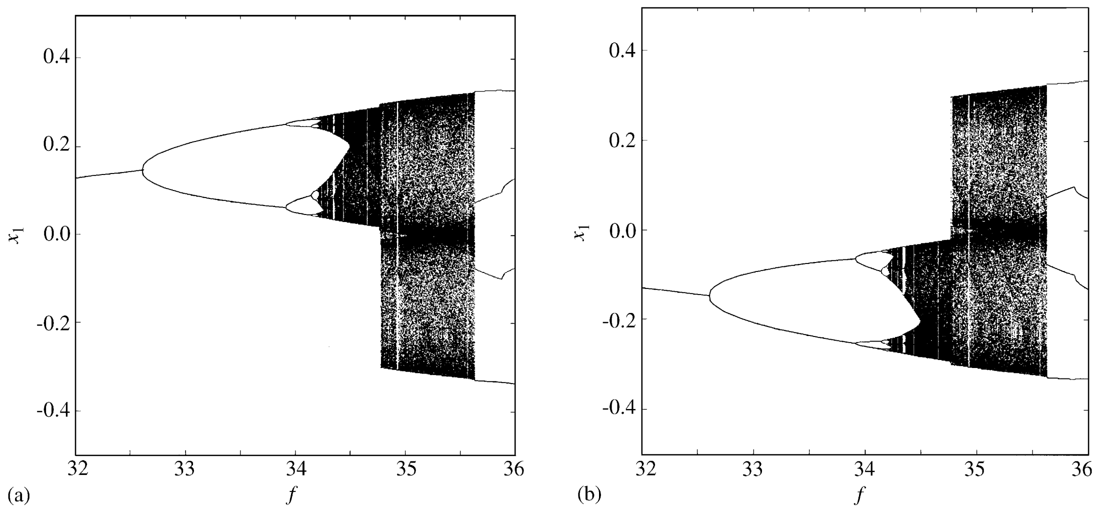


Fig 1. Bifurcation diagram for $x_1$ in the range $32 \leqslant f \leqslant 36$: (a) motion studied in Ref. [1] and (b) motion obtained with other starting values.

*Tel.: +32-2-629-35-66; fax: +32-2-629-28-65.
*E-mail address:* rvdooren@vub.ac.be (R. Van Dooren).

0022-460X/03/$ - see front matter 
doi:10.1016/S0022-460X(03)00343-2

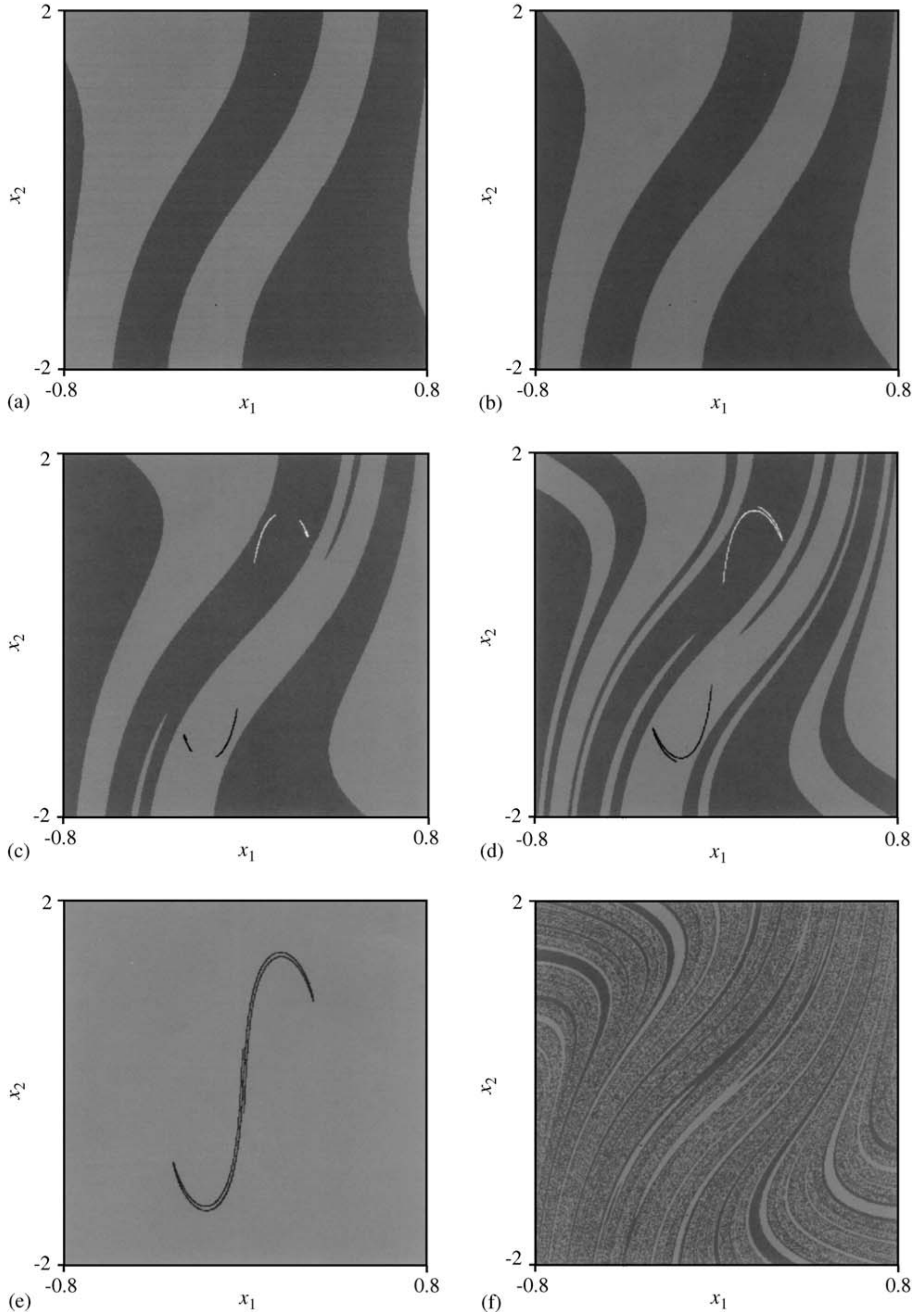


Fig. 2. Metamorphoses of the domains of attraction in the phase plane $x_1x_2$: (a) $f = 32$ (coexistence of two periodic attractors both having the period $1T$; basins are in light- and dark-gray); (b) $f = 33$ (two periodic attractors with the period $2T$); (c) $f = 34.3$ (two chaotic attractors indicated in white and in black); (d) $f = 34.6$ (two chaotic attractors); (e) $f = 35$ (the S-shaped chaotic attractor); $f = 36$ (two coexisting periodic attractors both with the period $4T$).

The numerical results of the dynamics of the gyro as represented in the bifurcation diagram in Fig. 2 in Ref. [1] with the parameter values $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$, are very inaccurate and also incomplete. In their bifurcation diagram with the amplitude $f$ in the range $32 \leqslant f \leqslant 36$, the transition from the $1T$-periodic solution (with $T = 2\pi/\omega$) to the period doubled $2T$-solution occurs for $f \approx 32.9$, the transition $2T \to 4T$ for $f \approx 34.35$, the transition $4T \to 8T$ for $f \approx 34.6$, the first appearance of the broadest zone of chaotic behavior for $f \approx 35.3$ and for $f \approx 36$ the behavior of the system is chaotic. However, the author found the following discrepancies. The transition $1T \to 2T$ takes place for $f \approx 32.57$, the transition $2T \to 4T$ for $f \approx 33.91$, the transition $4T \to 8T$ for $f \approx 34.14$ and the broadest chaos zone starts near $f \approx 34.8$. For $f \approx 36$ the author obtained $4T$-solutions. The corresponding bifurcation diagram is here given in Fig. 1(a). These results were found with the use of the package DYNAMICS [2]. They have been checked independently by applying the numerical method described in Ref. [3].

In addition, starting with another set of initial conditions for the numerical integration of system (1), the author obtained a second sequence of period doubling solutions. These results are illustrated in Fig. 1(b). The complete bifurcation diagram consists of the conjunction of Figs. 1(a) and (b). The occurrence of the second cascade of period doubling solutions is confirmed by a study of the basins of attraction of coexisting periodic or chaotic attractors. The metamorphoses of the basins of attraction in the phase plane $x_1x_2$ with a $400 \times 400$ grid of pixels are summarized in Fig. 2. For $f = 32$ Fig. 2(a) shows two domains of attraction (represented in light- and dark-gray) corresponding to two periodic attractors both with the period $1T$. For $f = 33$ (Fig. 2(b)) the period of the two coexisting attractors has changed to $2T$. With $f = 34.3$ (Fig. 2(c)) the behavior becomes chaotic whereby each attractor is generated by two parts. The chaotic attractors are represented by clusters of black and white dots. For $f = 34.6$ (Fig. 2(d)) the two parts in each chaotic attractor merge together. For $f = 35$ (Fig. 2(e)) the two chaotic attractors merge into one S-shaped attractor. Fig. 2(f) for $f = 36$ illustrates the basins of the two periodic attractors each with the period $4T$. These basins are highly fractal.

Finally, the behavior of the gyro in the vicinity of $f = 36$ is described. At the end of the broadest zone of chaotic behavior, i.e., at $f \approx 35.65$, a $4T$-solution appears which is symmetric. At $f \approx 35.87$ the symmetry of this solution is broken and two coexisting asymmetric solutions with the same period $4T$ occur. With $f = 36$ the co-ordinates $(x_1, x_2)$ of the Poincaré section points at $t = 0$ are: (0.33580, 0.59994), (−0.13232, −1.44114), (−0.32918, −0.68237), (0.07300, 1.25854) for the first solution. The co-ordinates of the second solution have opposite signs. A period doubling cascade is found with the next transition $4T \to 8T$ for $f \approx 36.04$. This sequence of period doubling solutions ends in the limit with chaotic behavior for $f \approx 36.1$.

## References

[1] H.-K. Chen, Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping, Journal of Sound and Vibration 255 (2002) 719–740.

[2] H.E. Nusse, J.A. Yorke, Dynamics: Numerical Explorations, 2nd Edition, Springer, New York, 1998.

[3] R. Van Dooren, Chaos in a pendulum with forced horizontal support motion: a tutorial, Chaos, Solitons and Fractals 7 (1996) 77–90.